Parameterized Approximation: A New Way of Looking at Parameterized Complexity and Some First Concrete Results

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Abstract: Parameterized complexity is basically a two-dimensional sequel to the one-dimensional framework of P versus NP (which is based on considering a single measurement of the input: its total size n). In parameterized algorithmics, a secondary measurement k is introduced to capture the effects of input structure or other considerations that affect computational complexity, and thus influence how efficient algorithms for a problem should be designed. This turns out to be almost universally relevant for real world input distributions. In P versus NP, the basic contrast is between polynomial runtimes $O(n^c)$ and runtimes that are $O(2^{n^c})$. In parameterized complexity, the basic contrast is between the two-variate function classes: FPT, meaning runtimes that are $f(k)n^c$, and runtimes that are $n^{g(k)}$ (the class XP). The usual tower of parameterized complexity classes is:

$$FPT \subseteq M[1] \subseteq W[1] \subseteq M[2] \subseteq W[2] \subseteq \cdots \subseteq XP$$

Perhaps surprisingly, there is a completely general way to consider FPT that refers us back to the fundamental (one-dimensional) notion of polynomial time. A parameterized problem Π is in FPT if and only there is a polynomial-time self-transformation from Π to Π (polynomial time in both n and k) that transforms (x, k) to (x', k'), where:

- (x, k) is YES if and only if (x', k') is YES.
- $k' \leq k$.
- $|x'| \leq g(k)$ for some univariate function g.

This point of view on FPT has been very productive in practical terms. This basically gives us a framework in which to investigate sophisticated polynomial-time pre-processing, an almost universally relevant coping strategy for hard problems, and because *kernelization* is P-time, *relevant even when the parameter k is* **not** *small.*

The talk is about a new area of research in parameterized algorithmics, where one can take a similarly general view of the whole ballgame: a completely different way of classifying the complexity of parameterized problems, bearing some similarity to the P-time kernelization view of the inside structure of FPT. We end up with a P-time approximation view of the total structure of the complexity of parameterized problems.

So far, there are only a few concrete results that are known about this alternate view of the parameterized universe. We will present one of these, and describe the few things that are known so far. Because this view is based on P-time algorithms, one could hope that, as with the P-time kernelization view of FPT, investigations in this direction may yield insights useful for practical algorithmics for *hard parameterized problems.*