Data Reduction for Domination in Graphs (2004; Alber, Fellows, Niedermeier)

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1 Problem Definition

The NP-complete Dominating Set problem is a notoriously hard problem:

Problem 1 (Dominating Set).
Input*: An undirected graph $G = (V, E)$ and an integer $k \geq 0$.*
Question*: Is there an $S \subseteq V$ with $|S| \leq k$ such that every vertex $v \in V$ is contained in S or has at least one neighbor in S?*

For instance, for an n-vertex graph its optimization version is known to be polynomial-time approximable only up to a factor of $\Theta(\log n)$ unless some standard complexity-theoretic assumptions fail [9]. In terms of parameterized complexity, the problem is shown to be W[2]-complete [8]. Although still NP-complete when restricted to planar graphs, the situation much improves here. In her seminal work, Baker showed that there is an efficient polynomial-time approximation scheme (PTAS) [6], and the problem also becomes fixed-parameter tractable [2, 4] when restricted to planar graphs. In particular, the problem becomes accessible to fairly effective data reduction rules and a kernelization result (see [16] for a general description of data reduction and kernelization) can be proven. This is the subject of this entry.

2 Key Results

The key idea behind the data reduction is preprocessing based on locally acting simplification rules. Exemplarily, here we describe a rule where the local neighborhood of each graph vertex is considered. To this end, we need the following definitions.

We partition the neighborhood $N(v)$ of an arbitrary vertex $v \in V$ in the input graph into three disjoint sets $N_1(v)$, $N_2(v)$, and $N_3(v)$ depending on local neighborhood structure. More specifically, we define

- $N_1(v)$ to contain all neighbors of v that have edges to vertices that are not neighbors of v;
- $N_2(v)$ to contain all vertices from $N(v) \setminus N_1(v)$ that have edges to at least one vertex from $N_1(v)$;
- $N_3(v)$ to contain all neighbors of v that are neither in $N_1(v)$ nor in $N_2(v)$.

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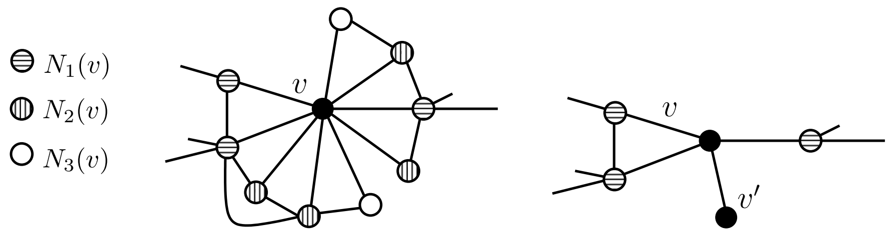

Figure 1: The left-hand side shows the partitioning of the neighborhood of a single vertex v. The right-hand side shows the result of applying the presented data reduction rule to this particular (sub)graph.

An example which illustrates such a partitioning is given in Figure 1 (left-hand side). A helpful and intuitive interpretation of the partition is to see vertices in $N_1(v)$ as *exits* because they have direct connections to the world outside the closed neighborhood of v, vertices in $N_2(v)$ as *guards* because they have direct connections to exits, and vertices in $N_3(v)$ as *prisoners* because they do not see the world outside $\{v\} \cup N(v)$.

Now consider a vertex $w \in N_3(v)$. Such a vertex only has neighbors in $\{v\} \cup N_2(v) \cup N_3(v)$. Hence, to dominate w, at least one vertex of $\{v\} \cup N_2(v) \cup N_3(v)$ *must* be contained in a dominating set for the input graph. Since v can dominate all vertices that would be dominated by choosing a vertex from $N_2(v) \cup N_3(v)$ into the dominating set, we obtain the following data reduction rule.

> If $N_3(v) \neq \emptyset$ for some vertex v, then remove $N_2(v)$ and $N_3(v)$ from G and add a new vertex v' with the edge $\{v, v'\}$ to G.

Note that the new vertex v' can be considered as a "gadget vertex" that "enforces" v to be chosen into the dominating set. It is not hard to verify the correctness of this rule, that is, the original graph has a dominating set of size k iff the reduced graph has a dominating set of size k. Clearly, the data reduction can be executed in polynomial time [5]. Note, however, that there are particular "diamond" structures which are not amenable to this reduction rule. Hence, a second, somewhat more complicated rule based on considering the joint neighborhood of *two* vertices has been introduced [5].

Altogether, the following core result could be shown [5].

Theorem 1. *A planar graph $G = (V, E)$ can be reduced in polynomial time to a planar graph $G' = (V', E')$ such that G has a dominating set of size k iff G' has a dominating set of size k and $|V'| = O(k)$.*

In other words, the theorem states that Dominating Set in planar graphs has a linear-size problem kernel. The upper bound on $|V'|$ was first shown to be $335k$ [5] and was then further improved to $67k$ [7]. Moreover, the results can be extended to graphs of bounded genus [10]. In addition, similar results (linear kernelization) have been recently obtained for the Full-Degree Spanning Tree problem in planar graphs [13]. Very recently, these results have been generalized into a methodological framework [12].

3 Applications

Dominating Set is considered to be one of the most central graph problems [14, 15]. Its applications range from facility location to bioinformatics.

4 Open Problems

The best lower bound for the size of a problem kernel for Dominating Set in planar graphs is $2k$ [7]. Thus, there is quite a gap between known upper and lower bounds. In addition, there have been some considerations concerning a generalization of the above discussed data reduction rules [3]. To what extent such extensions are of practical use remains to be explored. Finally, a study of deeper connections between Baker's PTAS results [6] and linear kernelization results for Dominating Set in planar graphs seems to be worthwhile for future research. Links concerning the class of problems amenable to both approaches have been detected recently [12]. The research field of data reduction and problem kernelization as a whole together with its challenges is discussed in a recent survey [11].

5 Experimental Results

The above described theoretical work has been accompanied by experimental investigations on synthetic as well as real-world data [1]. The results have been encouraging in general. However, note that grid structures seem to be a hard case where the data reduction rules remained largely ineffective.

6 Cross References

Entry 00529 (Connected Dominating Set); Entry 00280 (Dominating Set).

7 Recommended Reading

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